Hessian矩阵正定推导1

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$\bar{F}_{2\times 2}$

$\bar{F} = U \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} V^T$

$\vec{m}_1 \quad \vec{m}_2$

$\bar{\lambda}_1 = |\bar{F} \cdot \vec{m}_1|$

$\bar{\lambda}_2 = |\bar{F} \cdot \vec{m}_2|$

$\Psi = \Psi(\lambda_1, \lambda_2) = \Psi(\bar{\lambda}_1, \bar{\lambda}_2)$ 凸函数

$\frac{\partial^2 \Psi}{\partial \lambda^2}$ Hessian 正定

$\Psi = \Psi(\lambda_1, \lambda_2) = \Psi(\bar{\lambda}_1, \bar{\lambda}_2) =$ 好量函数 $w_1(\bar{\lambda}_1) + w_2(\bar{\lambda}_2)$

$$\frac{\partial \Psi}{\partial \lambda_1} = \frac{\partial w_1}{\partial \lambda_1} + \frac{\partial w_2}{\partial \lambda_1}$$

$$= w_1' \frac{\partial \bar{\lambda}_1}{\partial \lambda_1} + w_2' \cdot \frac{\partial \bar{\lambda}_2}{\partial \lambda_1}$$

$$\frac{\partial^2 \Psi}{\partial \lambda_1^2} = w_1'' \left(\frac{\partial \bar{\lambda}_1}{\partial \lambda_1}\right)^2 + w_1' \frac{\partial^2 \bar{\lambda}_1}{\partial \lambda_1^2}$$

$$+ w_2'' \left(\frac{\partial \bar{\lambda}_2}{\partial \lambda_1}\right)^2 + w_2' \frac{\partial^2 \bar{\lambda}_2}{\partial \lambda_1^2}$$

$$\frac{\partial^2 \Psi}{\partial \lambda_1 \partial \lambda_2} = w_1'' \frac{\partial \bar{\lambda}_1}{\partial \lambda_1} \cdot \frac{\partial \bar{\lambda}_1}{\partial \lambda_2} + w_1' \frac{\partial^2 \bar{\lambda}_1}{\partial \lambda_1 \partial \lambda_2}$$

$$+ w_2'' \frac{\partial \bar{\lambda}_2}{\partial \lambda_1} \frac{\partial \bar{\lambda}_2}{\partial \lambda_2} + w_2' \frac{\partial^2 \lambda_2}{\partial \lambda_1 \partial \lambda_2}$$

$$\begin{cases} \dfrac{\partial \bar{\lambda}_1}{\partial \lambda_1} & \dfrac{\partial \bar{\lambda}_1}{\partial \lambda_2} \\ \dfrac{\partial^2 \bar{\lambda}_1}{\partial \lambda_1^2} & \dfrac{\partial^2 \bar{\lambda}_1}{\partial \lambda_1 \partial \lambda_2} \end{cases}$$