

## 线性材料能量

$$\psi = \frac{1}{2} \sigma^T \varepsilon \quad \sigma = C \varepsilon \quad \varepsilon = \frac{1}{2} (F^T + F) \cdot \mathbf{1}$$

↓

$$\psi = \frac{1}{2} \varepsilon^T C \varepsilon$$

$$C_{ortho} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ 2\sigma_{12} \\ 2\sigma_{23} \\ 2\sigma_{31} \end{bmatrix} = C \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \end{bmatrix}$$

$$A = \gamma \begin{bmatrix} E_1 (1 - \nu_{23}\nu_{32}) & E_2 (\nu_{12} + \nu_{23}\nu_{31}) & E_3 (\nu_{13} + \nu_{12}\nu_{23}) \\ E_1 (\nu_{21} + \nu_{21}\nu_{33}) & E_2 (1 - \nu_{13}\nu_{31}) & E_3 (\nu_{23} + \nu_{21}\nu_{13}) \\ E_1 (\nu_{31} + \nu_{31}\nu_{22}) & E_2 (\nu_{32} + \nu_{12}\nu_{31}) & E_3 (1 - \nu_{12}\nu_{21}) \end{bmatrix}$$

$$B = \begin{bmatrix} \nu_{12} & 0 & 0 \\ 0 & \nu_{23} & 0 \\ 0 & 0 & \nu_{31} \end{bmatrix} \quad \gamma = \frac{1}{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13}}$$

## 共轭线性

$$F = RS$$

$$\varepsilon = S - \mathbf{I}$$

$$\psi(S) = \frac{1}{2} (S - \mathbf{I})^T C (S - \mathbf{I})$$

$$= \frac{1}{2} (S^T C S - 2 C S \mathbf{I} - \mathbf{I} C \mathbf{I})$$

$$RS = U \Sigma V^T = U V^T V \bar{\Sigma} V^T$$

$$S = V \bar{\Sigma} V^T$$

$$\psi(\bar{\Sigma}) \quad \text{巨复杂!!!}$$

$$\boxed{\varepsilon = \bar{\Sigma} - \mathbf{I} \quad \text{去简化?}}$$

$$\psi(\bar{\Sigma}) = \frac{1}{2} (\bar{\Sigma} C \bar{\Sigma} - 2 C \bar{\Sigma} \mathbf{I} - \mathbf{I} C \mathbf{I})$$

$$= 2r \cdot \{$$

$$E_1 (1 - \nu_{23}\nu_{32}) \lambda_1^2 - 2E_1 (1 - \nu_{23}\nu_{32}) \lambda_1 - 2E_1 (\nu_{21} + \nu_{21}\nu_{33}) \lambda_1 - 2E_1 (\nu_{31} + \nu_{31}\nu_{22}) \lambda_1$$

+

$$E_2 (1 - \nu_{13}\nu_{31}) \lambda_2^2 - 2E_2 (1 - \nu_{13}\nu_{31}) \lambda_2 - 2E_2 (\nu_{12} + \nu_{12}\nu_{23}) \lambda_2 - 2E_2 (\nu_{32} + \nu_{12}\nu_{31}) \lambda_2$$

+

$$E_3 (1 - \nu_{12}\nu_{21}) \lambda_3^2 - 2E_3 (1 - \nu_{12}\nu_{21}) \lambda_3 - 2E_3 (\nu_{13} + \nu_{12}\nu_{23}) \lambda_3 - 2E_3 (\nu_{32} + \nu_{12}\nu_{31}) \lambda_3$$

+

$$[E_1 (\nu_{21} + \nu_{21}\nu_{33}) + E_2 (\nu_{12} + \nu_{12}\nu_{23})] \lambda_1 \lambda_2$$

+

$$[E_1 (\nu_{31} + \nu_{31}\nu_{22}) + E_3 (\nu_{13} + \nu_{12}\nu_{23})] \lambda_1 \lambda_3$$

+

$$[E_2 (\nu_{32} + \nu_{12}\nu_{31}) + E_3 (\nu_{32} + \nu_{12}\nu_{31})] \lambda_2 \lambda_3 \quad \}$$

↓

$$\psi(\bar{\Sigma}) = \psi_{iso}(\bar{\Sigma}) + \psi_{ortho}(\bar{\Sigma})$$